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The effect of adiabatic wall roughness elements on natural convection heat transfer in vertical enclosures

M. RUHUL AMIN

Department of Mechanical Engineering, 220 Roberts Hall, Montana State University, Bozeman, MT 59717, U.S.A.

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Abstract—The natural convection heat transfer in a two-dimensional vertical enclosure fitted with a periodic array of large rectangular elements on the bottom horizontal wall is investigated numerically. The numerical investigation is carried out for the ranges of Rayleigh numbers (Ra_H) and aspect ratios (W/H) of 1.85×10^2–1.85×10^7 and 0.4–2.4, respectively. Prandtl numbers (Pr) of 0.72 and 4.52 are considered. It is found that the large roughness elements reduce the heat transfer rate across the enclosure. The reduction in heat transfer is more significant for enclosures with $W/H > 1$. The highest value of heat transfer reduction obtained in this research is 44%. This is obtained for an enclosure with $W/H = 2.4$ at $Ra_H = 1.85\times10^4$ and $Pr = 4.52$.

INTRODUCTION

DUE TO its inherent reliability, natural convection heat transfer has become one of the most important modes of cooling in electronic equipment. Depending on their mounting configurations and end conditions electronic circuit cards often form vertical channels, or complete enclosures. If the surfaces of such circuit cards are roughened by roughness elements, it can affect the heat transfer rates from the cards. Therefore, a fundamental understanding of the mechanisms of the natural convection heat transfer in arrays of circuit cards with rough surfaces could lead to the more efficient design of cooling systems for electronic equipment. The ability to predict the effects of roughness elements on natural convection heat transfer in enclosures can also lead to an efficient design of solar collectors and building energy systems. The natural convection flows in a differentially heated enclosure with wall roughness elements have also some other applications such as fluid-filled thermal storage tanks, safety and operation of nuclear reactor, and prevention and spread of fire in buildings.

Anderson and Bohn [1] performed an experimental investigation of the effects of large surface roughness elements on the natural convection heat transfer in a cubical enclosure filled with water. One vertical wall was roughened by adding distributed roughness elements, which consisted of a series of intersecting grooves rotated 45° from the horizontal. They used both isothermal and constant heat flux boundary conditions for the rough vertical wall and covered the range of Rayleigh numbers and modified Rayleigh numbers from 7.7×10^9 to 7.3×10^{10} and 6.8×10^{11} to 9.2×10^{12}, respectively. Their experiment showed that, due to the surface roughness elements on the isothermal heated wall, the overall Nusselt number for the enclosure was increased by 15% from that of a corresponding smooth-walled enclosure at a Rayleigh number of 3.3×10^{10}. Shakerin and Loehrke [2] and Shakerin *et al.* [3] performed a numerical and experimental study of the effects of roughness elements on natural convection in a vertical enclosure bounded by two isothermal vertical walls and two adiabatic horizontal walls. Either one or two roughness elements of boundary layer thickness height were mounted on the hot vertical wall. The authors concluded that for an enclosure with an aspect ratio of 1.0, filled with a fluid with a Prandtl number of 0.7, and at a Rayleigh number of 10^6, the presence of roughness elements increased the overall Nusselt number of the enclosure by 12% from that of a corresponding smooth-walled enclosure. A numerical and experimental study of the effects of roughness elements on natural convection in a vertical enclosure bounded by two isothermal vertical walls and two horizontal adiabatic walls was performed in ref. [4]. A periodic array of large roughness elements was mounted on the hot vertical wall in such a manner that the volume of the fluid in the enclosure remained the same as that of the corresponding smooth-walled enclosure. Both sinusoidal and rectangular roughness elements were considered for enclosures with aspect ratios (width/height) of 0.4 and 1.0, with a range of Grashof numbers from 1.55×10^3 to 6.64×10^6. The Prandtl numbers were 0.72 and 4.52. The greatest enhancement found in ref. [4] was 35.5%; which was obtained at a Grashof number of 4.07×10^6 and a Prandtl number of 4.52 with an enclosure aspect ratio of 0.4, containing four sinusoidal roughness elements.

Nansteel and Grief [5] performed an experimental investigation of the natural convection heat transfer

NOMENCLATURE

A^*	amplitude of roughness element, see Fig. 1
A	dimensionless amplitude of roughness element, A^*/H
C_p	constant pressure specific heat of fluid
g	magnitude of acceleration due to gravity
H	height of the enclosure
k	thermal conductivity of fluid
$\overline{Nu}_R$	average Nusselt number of an enclosure containing roughness elements
$\overline{Nu}_S$	average Nusselt number of a smooth-walled enclosure
P^*	motion pressure
P	dimensionless motion pressure, $P^*H^2/\rho v^2$
Pr	Prandtl number, $\mu C_p/k$
Ra_H	Rayleigh number, $g\beta(T_H^*-T_C^*)H^3/v\alpha$
T^*	temperature
T	dimensionless fluid temperature, $(T^*-T_C^*)/(T_H^*-T_C^*)$
T_H^*	hot wall temperature
T_C^*	cold wall temperature
u^*	x^*-component of velocity
u	dimensionless x-component of velocity, u^*H/v
v^*	y^*-component of velocity
v	dimensionless y-component of velocity, v^*H/v
W	width of enclosure
x^*, y^*	spatial coordinates, see Fig. 1
x, y	dimensionless spatial coordinates, x^*/H, y^*/H
$\bar{x}^*, \bar{y}^*$	local coordinates for rectangular roughness elements, see Fig. 2
$\bar{x}, \bar{y}$	dimensionless local coordinates for rectangular roughness elements, $\bar{x}^*/H$, $\bar{y}^*/H$
y_L^*	y^*-coordinate of the bottom adiabatic wall
y_L	y-coordinate of the bottom adiabatic wall, y_L^*/H.

Greek symbols

α	thermal diffusivity
β	coefficient of thermal expansion of fluid
δ^*	period of roughness elements
δ	dimensionless period of roughness elements, δ^*/H
μ	dynamic viscosity of fluid
v	kinematic viscosity of fluid
ρ	density of fluid
ϕ^*	phase shift of rectangular roughness elements
ϕ	dimensionless phase shift of rectangular roughness elements, ϕ^*/H
ψ	dimensionless stream function, defined by equations (13) and (14).

in an enclosure with aspect ratio, $W/H = 2$, fitted with a partial partition hanging from the top adiabatic horizontal wall. They conducted experiments with $Pr = 3.5$, for a range of Rayleigh numbers from 2.3×10^{10} to 1.1×10^{11}. Nansteel and Grief [5] concluded that the partial partition reduces the heat transfer rate across the enclosure. Chang *et al.* [6] performed a numerical investigation for an enclosure with an aspect ratio of 1 and fitted with partitions from top and bottom adiabatic walls. They performed the investigation in the range of Grashof numbers of 10^3–10^8. Chang *et al.* [6] concluded that the partitions on the adiabatic walls reduce the overall heat transfer across the enclosure. As a follow-up work, Bajorek and Lloyd [7] performed an experimental investigation in an enclosure having the same geometry as in ref. [6]. The authors concluded that the partitions on the adiabatic walls reduce the local Nusselt number on the hot wall. Zimmerman and Acharya [8] performed a numerical study with a geometry the same as that reported in ref. [6]. However, in ref. [6] the horizontal end walls of the enclosure were assumed to be perfectly adiabatic, but in ref. [8], the horizontal walls were considered to be perfectly conducting. Zimmerman and Acharya concluded that the average Nusselt number of the enclosure is significantly reduced due to the presence of the baffles except at low Rayleigh numbers.

Kaviany [9] made a numerical study of the effects on natural convection of a semi-cylindrical protuberance, which was located on the adiabatic bottom wall of an otherwise square cavity with isothermal vertical walls maintained at different temperatures and adiabatic horizontal walls. He performed the study for a fluid with a Prandtl number of 0.71 and a range of Rayleigh numbers up to 10^4. He concluded that the protuberance causes a decrease in the local heat transfer rate on the lower portion of the cold vertical wall. Bilski *et al.* [10] performed an experimental investigation on the laminar natural convection flows in square and partially partitioned enclosures with two vertical isothermal walls and two horizontal insulated walls for a range of Rayleigh numbers from 10^5 to 10^6 and compared their results with those of numerical studies. Lin and Bejan [11] performed an experimental and analytical study of the effect of a partial partition protruding from the bottom adiabatic wall of a rectangular enclosure. Their study was conducted in the Rayleigh number range of 10^9–10^{10} and for an aspect ratio $(W/H) = 2$. The authors concluded that an increase in height of the partition drastically reduces the heat transfer rate across the enclosure. Most recently Acharya and Jetli [12] performed a numerical study for the heat transfer in square enclosure fitted with a partial partition on the bottom horizontal wall. They performed the study in the range of Rayleigh numbers from 10^5 to 10^6 and assumed that the horizontal end walls of the enclosure were perfectly conducting. Acharya and Jetli concluded that the heat transfer rate across the enclosure is reduced as the height of the partition is increased. Little change in heat transfer rate was observed when the location of the partition on the horizontal wall was changed.

The objective of the present study is to determine the effects of mounting a periodic array of large rectangular roughness elements on the bottom horizontal wall of a two-dimensional rectangular enclosure filled with a Newtonian fluid. Two basic geometries as shown in Fig. 1 are investigated. Each enclosure is assumed to be completely filled with the same Newtonian fluid. The vertical walls of the enclosures are isothermal surfaces with the left-hand vertical wall of each enclosure maintained at a higher temperature, T_H^*, than the temperature of the right-hand vertical wall, T_C^*. The horizontal walls of each enclosure are adiabatic surfaces.

The enclosure shown in Fig. 1(a) has smooth walls. The natural convection flows in this enclosure have been the subject of many studies. This enclosure is used in the present research to study the effects of mounting a large roughness element on the bottom horizontal wall of the enclosure. The effects are studied by comparing the natural convection flows in the rough-walled enclosure with that of a corresponding smooth-walled enclosure. It should be noted that each enclosure with roughness elements is designed such that the volume of fluid in this enclosure is the same as the volume of fluid in the corresponding enclosure with smooth walls and the same aspect ratio. This is necessary for the purpose of this investigation since attaching large roughness elements to a horizontal wall can appreciably reduce the volume of fluid in the enclosure from that contained in the initially smooth-walled enclosure. In that situation, it would be impossible to determine whether the change in heat transfer rate across the rough-walled enclosure was caused due to the presence of the roughness elements or due to the decrease in the fluid volume or both.

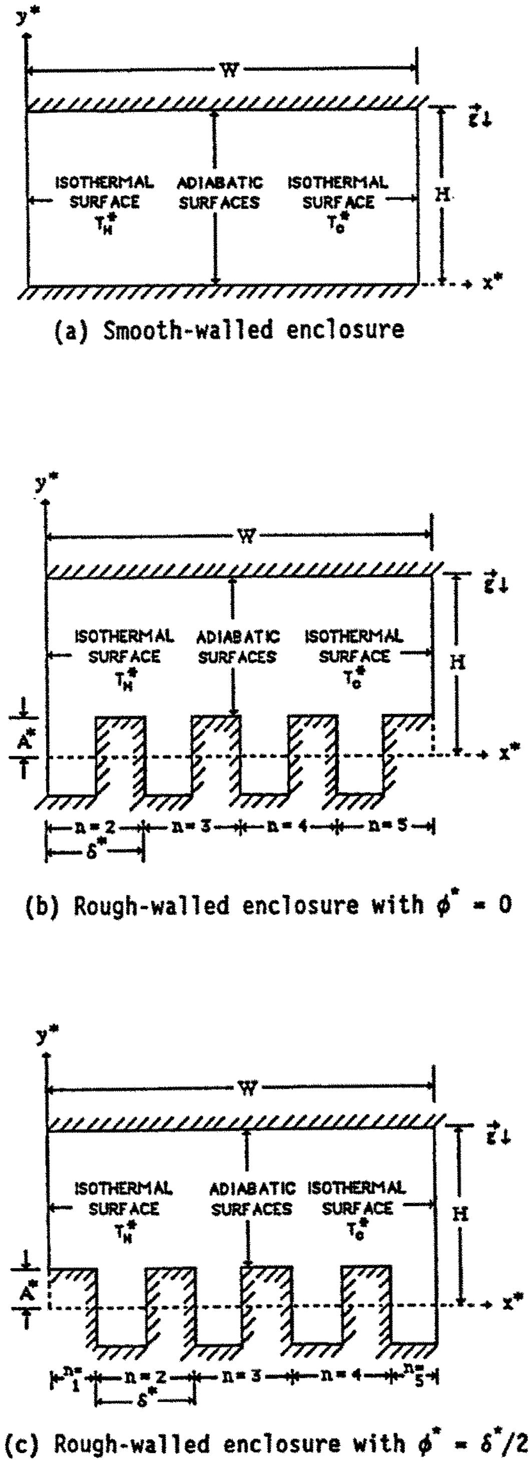

FIG. 1. Investigated enclosure geometries.

MATHEMATICAL FORMULATION

Modeling assumptions

In the present work it is assumed that the natural convection flows of the Newtonian fluids in the enclosures of interest are steady-state, two-dimensional, and laminar. It is also assumed that the volume of the fluid in the rough-walled enclosure is the same as that of the smooth-walled enclosure, that the Boussinesq approximation is valid, that the fluid is a radiatively non-participating medium, and that viscous dissipation and compressive work are negligibly small.

As shown in ref. [4], the requirement of keeping the amount of fluid in the rough-walled enclosure equal to that of the smooth-walled enclosure may be determined by inspection of Fig. 2, which shows the detailed geometry of one period of a rectangular

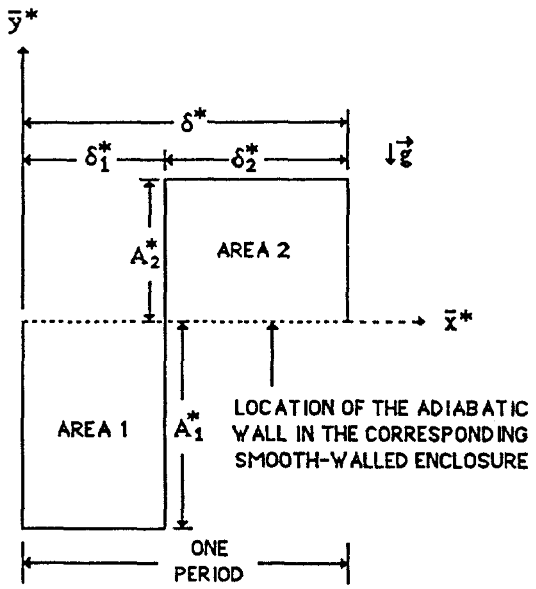

FIG. 2. Basic geometry of a single roughness element shown in local coordinates.

roughness element. It can be seen from Fig. 2 that to satisfy this requirement, area 1 must be equal to area 2, or

$$A_1^*\delta_1^* = A_2^*\delta_2^*. \tag{1}$$

In general, the distances δ_1^* and δ_2^* may not be equal. In this research the amplitudes A_1^* and A_2^* are assumed to be equal, that is

$$A_1^* = A_2^* \equiv A^*. \tag{2}$$

This reduces equation (1) to

$$\delta_1^* = \delta_2^* = \delta^*/2. \tag{3}$$

It was also assumed that (W/δ^*) is always an integer. This integer is defined as I_{PER}.

The governing equations

Based on the above modeling assumptions the non-dimensional governing equations for the conservation of mass, momentum, and energy are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4a}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \tag{4b}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{Ra_H}{Pr}T \tag{4c}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{Pr}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{4d}$$

where, P, the "motion pressure" is defined as the actual pressure in the fluid less the pressure when the fluid is at rest at the reference temperature. The temperature of the cold vertical wall of the enclosure, T_C^*, serves as the reference temperature in the buoyancy force term in equation (4c).

The corresponding boundary conditions are:

for $(-A \leqslant y \leqslant 1)$

$$u(0,y) = v(0,y) = 0 \tag{5a}$$

$$T(0,y) = 1; \tag{5b}$$

for $(0 \leqslant x \leqslant W/H)$

$$u(x,1) = v(x,1) = 0 \tag{6a}$$

$$\frac{\partial T(x,1)}{\partial y} = 0; \tag{6b}$$

for $(-A \leqslant y \leqslant 1)$

$$u(W/H,y) = v(W/H,y) = 0 \tag{7a}$$

$$T(W/H,y) = 0; \tag{7b}$$

for $(0 \leqslant x \leqslant W/H)$

$$u(x,y_L) = v(x,y_L) = 0 \tag{8a}$$

$$\frac{\partial T(x,y_L)}{\partial y} = 0 \tag{8b}$$

(for the horizontal part of the roughness elements)

$$\frac{\partial T(x,y_L)}{\partial x} = 0 \tag{8c}$$

(for the vertical part of the roughness elements).

For the smooth-walled enclosure

$$y_L = 0. \tag{9}$$

Following the method in ref. [4], y_L for the rough-walled enclosures (Figs. 1(b) and (c)) can be expressed for each period of the roughness element array by the following set of equations:

$(-A \leqslant y_L \leqslant 0)$ for $\bar{x} = 0$

(for all i, except $i = 2$ when $\phi = 0$) (10a)

$y_L = -A$ for $\bar{x} = 0$ (when $i = 2$ and $\phi = 0$) (10b)

$y_L = -A$ for $(0 < \bar{x} < \delta/2)$ (10c)

$(-A \leqslant y_L \leqslant A)$ for $\bar{x} = \delta/2$ (for all i, except $i = I_{PER}+1$ when $\phi = \delta/2$) (10d)

$y_L = -A$ for $\bar{x} = \delta/2$ (when $i = I_{PER}+1$ and $\phi = \delta/2$) (10e)

$y_L = A$ for $(\delta/2 < \bar{x} < \delta)$ (10f)

and

$(0 \leqslant y_L \leqslant A)$ for $\bar{x} = \delta$. (10g)

The non-dimensional x-coordinate for the bottom adiabatic wall is expressed as

$$x = \bar{x} + (i-2)\delta + \phi. \tag{11}$$

In the above equations, i, is a positive integer that identifies a specific period in the array and is defined such that it assumes a value of 1 for $x \leqslant \phi$ and then increases in increments of 1 for each period traversed in moving across the wall roughness element array until it reaches a maximum possible value of $i_{max} = (I_{PER}+1)$.

The non-dimensional form of the stream function can be expressed as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \tag{12}$$

where, the stream function ψ is defined by

$$u = \frac{\partial \psi}{\partial y} \tag{13}$$

$$v = -\frac{\partial \psi}{\partial x}. \tag{14}$$

The impermeability and zero tangential velocity slip on the solid walls require that

$$\psi = \partial \psi / \partial n = 0, \text{ on the boundary.} \tag{15}$$

Heat transfer

The overall heat transfer across the enclosure is expressed in terms of the average Nusselt number, which is defined in non-dimensional form as

$$\overline{Nu} = \int_{A}^{1} -\frac{\partial T(0,y)}{\partial x}\,\mathrm{d}y = \int_{A}^{1} -\frac{\partial T(W/H,y)}{\partial x}\,\mathrm{d}y \tag{16a}$$

for $\phi = 0$ (Fig. 1(b)), and

$$\overline{Nu} = \int_{A}^{1} -\frac{\partial T(0,y)}{\partial x}\,\mathrm{d}y = \int_{-A}^{1} -\frac{\partial T(W/H,y)}{\partial x}\,\mathrm{d}y \tag{16b}$$

for $\phi = \delta/2$ (Fig. 1(c)). The above integrals are evaluated by using Simpson's rule.

NUMERICAL PROCEDURE

The governing equations for the present research were solved by using the code NACHOS II, developed by Gartling [13]. This code is a general purpose finite element code designed for the solution of both transient and steady-state two-dimensional laminar flows that are governed by Navier–Stokes equations. An earlier version of this code was successfully used by different investigators [4, 14, 15], and fairly accurate results were obtained. As a test problem for the accuracy of the NACHOS II code, several cases were executed at different Rayleigh numbers for a smooth-walled enclosure (Fig. 1(a)) with an aspect ratio $(W/H) = 1.0$, and $Pr = 10.0$. The computed values of the average Nusselt number $(\overline{Nu}_S)$ for these cases were compared with the corresponding results of Elder [16], de Vahl Davis [17], and MacGregor and Emery [18].

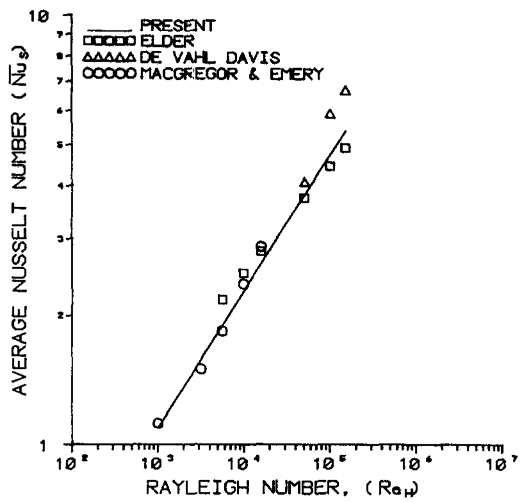

FIG. 3. Comparison of the results computed by the present method with the others for a smooth-walled enclosure with $W/H = 1.0$.

This comparison is shown in Fig. 3 from which it can be seen that the results obtained by NACHOS II code is in good agreement with the others.

The non-dimensional forms of the governing equations and boundary conditions of the present research show that the average Nusselt number of the smooth-walled enclosure $(\overline{Nu}_S)$ and rough-walled enclosure $(\overline{Nu}_R)$ have the following functional dependencies:

$$\overline{Nu}_S = \overline{Nu}_S[Ra_H, Pr, W/H] \tag{17a}$$

$$\overline{Nu}_R = \overline{Nu}_R[Ra_H, Pr, W/H, A, \delta, \phi]. \tag{17b}$$

In this research, the effects of all the parameters in equations (17) were explored.

The finite element grid used by the NACHOS II were different for different geometries of the enclosures. The elements were arranged such that more of them were packed into regions of large gradients of velocity and/or temperature. The grids were generated by the internal grid generator of the code. It should be noted here that the grid generator of the code can generate nine grid points for each element. These grid points are distributed as follows: four at the corner points of the element, one at the center of each side of the element (total of four), and one grid point at the center of the element. A typical example of the element distribution for a particular geometry is shown in Fig. 4.

RESULTS AND DISCUSSION

In this research, all the governing non-dimensional parameters were varied to study the effect of roughness elements on natural convection heat transfer across the enclosure. The ranges of Rayleigh numbers (Ra_H), aspect ratios (W/H), and roughness element periods (δ) considered were 1.85×10^2–1.85×10^7, 0.4–2.4, and 0.2–0.8, respectively. Results are presented for Prandtl numbers (Pr) of 0.72 and 4.52. Rect-

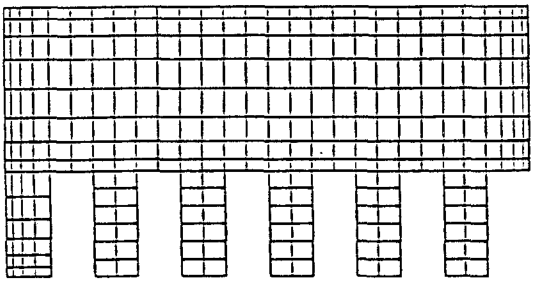

FIG. 4. Typical non-uniform computational grid for an enclosure with $W/H = 2.4$, $A = 0.24$, $\delta = 0.4$, $\phi = 0$.

angular roughness element amplitudes (A) of 0.12 and 0.24, and phase shifts (ϕ) of 0 and $\delta/2$ were considered in this study.

The rate of heat transfer by natural convection flow in a vertically-oriented, smooth-walled enclosure was determined to study the effect of placing large roughness elements in a corresponding rough-walled enclosure. The smooth-walled enclosures have been the subject of study by various authors. The physics of natural convection flows in these types of enclosures are well understood [19, 20], and will not be discussed here.

As reported earlier, the effect of adding large rectangular roughness elements was determined by comparing the overall Nusselt number of a rough-walled enclosure with that of a corresponding smooth-walled enclosure. The results are reported as the ratio of the average Nusselt numbers of the rough and smooth-walled enclosures, $[\overline{Nu}_R/\overline{Nu}_S]$. All the six non-dimensional parameters that control the governing equations were varied and a total of 60 cases were executed for the rough-walled enclosure. Figures 5–7 show streamlines and isotherms for an enclosure with $W/H = 2.4$, $A = 0.24$, $\delta = 0.4$, $\phi = 0$ at $Pr = 4.52$, and for $Ra_H = 1.85 \times 10^2$, 1.85×10^4, and 1.85×10^6, respectively. The arrows show the locations of the corresponding smooth-walled enclosures. The temperature profiles across the width of such an enclosure, at $y = 0.62$, for various values of Ra_H are shown in Fig. 8. These figures distinctly show the characteristics of the conduction regime at low Rayleigh numbers, boundary-layer flow regime at high Rayleigh numbers, and transition regime in between the two. In all the cases executed for the rough-walled enclosure in this research, the heat transfer rates decreased (i.e. Nusselt number ratio less than 1) except for a few boundary-layer flow regime cases where the heat transfer rates approximately remained the same as that of the corresponding smooth-walled enclosure.

Figure 9 shows that as the aspect ratio of the enclosure is increased from 0.4 to 2.4, the ratio of the Nusselt numbers is decreased. This is due to the fact that enclosures with a larger aspect ratio have a larger portion of their bottom wall roughened. Thus the effect of roughness elements, which is to reduce the heat transfer rate, is greater for enclosures with larger aspect ratio. Figure 9 also shows that over the range of Rayleigh numbers investigated for the enclosure with an aspect ratio of 2.4, the ratio of Nusselt numbers, $[\overline{Nu}_R/\overline{Nu}_S]$, first decreases and then increases with increasing Rayleigh number. The reason for this trend can be understood by examining Figs. 5–7. It can be seen from these figures that only in the convective regime (high Rayleigh number), the streamlines penetrate the roughness elements and hence exhibits an increase in Nusselt number. For the other regimes, there is essentially no flow in the cavity created by the roughness element and the left wall, so there is little heat transfer here. Since the average Nusselt number ($\overline{Nu}_R$) is based on the heat transfer from the entire left wall, and not just the active area, $\overline{Nu}_R$ is reduced for the cases where part of the left wall is inactive due to the presence of the roughness element. For enclosures with aspect ratios of 0.4 and 1.0 the ratios of Nusselt numbers increase with increasing Rayleigh number except at $Ra_H = 1.85 \times 10^7$. The lowest value of the Nusselt number ratio obtained in this research is 0.56, that is, 44% reduction in the heat transfer rate. This is obtained for an enclosure with $W/H = 2.4$, $A = 0.24$, $\delta = 0.4$, $\phi = 0$ at $Pr = 4.52$, $Ra_H = 1.85 \times 10^4$ and can be seen in Fig. 10.

The effect of phase shift of the roughness elements on the ratios of Nusselt numbers can be seen in Fig. 10. In this figure curve A is compared with curve B, curve C is compared with curve D, and curve E is compared with curve F to see the effect of phase

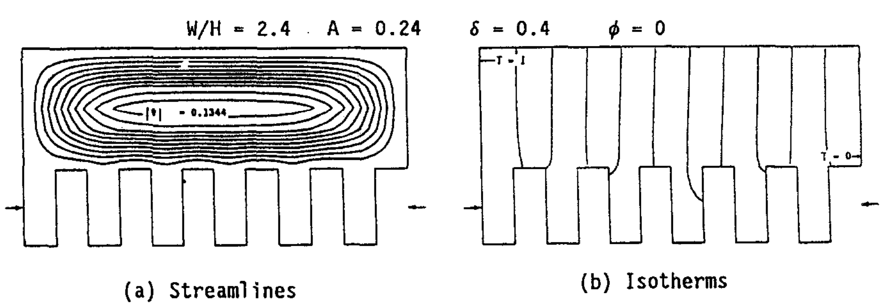

FIG. 5. Computed streamlines and isotherms in a rough-walled enclosure for $Ra_H = 1.85 \times 10^2$, $Pr = 4.52$.

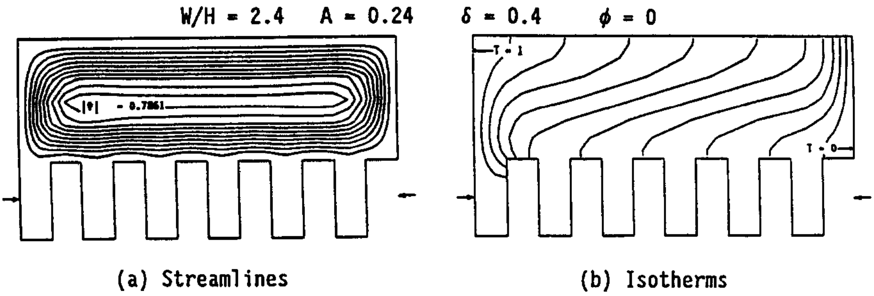

FIG. 6. Computed streamlines and isotherms in a rough-walled enclosure for $Ra_H = 1.85 \times 10^4$, $Pr = 4.52$.

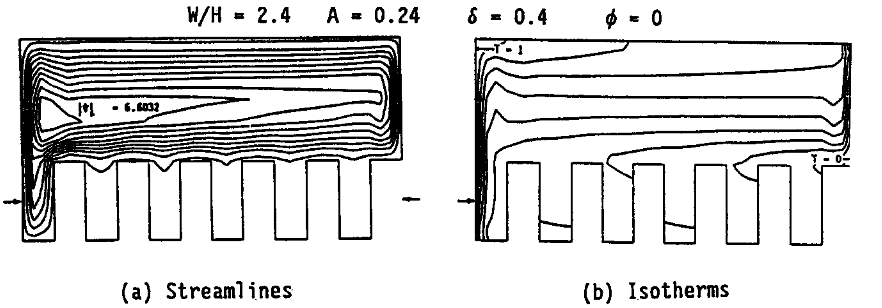

FIG. 7. Computed streamlines and isotherms in a rough-walled enclosure for $Ra_H = 1.85 \times 10^6$, $Pr = 4.52$.

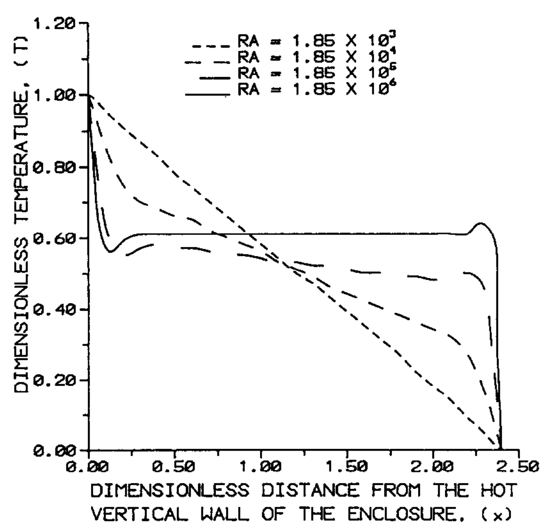

FIG. 8. The effect of Rayleigh number on the horizontal temperature distribution at $y = 0.62$ in a rough-walled enclosure with $W/H = 2.4$, $A = 0.24$, $\delta = 0.4$, $\phi = 0$.

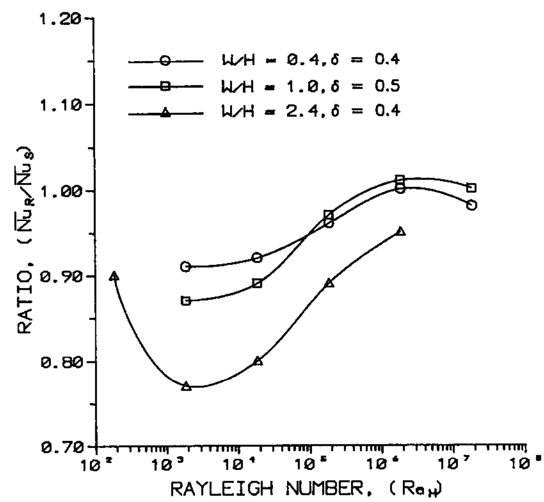

FIG. 9. The effect of aspect ratio on the ratio of Nusselt numbers for an enclosure with $A = 0.12$, $\phi = 0$, and $Pr = 4.52$.

shift. For each of these pairs, the Prandtl number, the enclosure aspect ratio, and the roughness element amplitude and period were kept constant while the phase shift was changed from 0 to $\delta/2$ (refer to Figs. 1(b) and (c)). It is observed that phase shift has little or no effect on the ratio of the Nusselt numbers for the enclosure in the conduction regime. Changing the phase shift from 0 to $\delta/2$ in the transition and boundary-layer regimes, however, reduces the ratio of the Nusselt numbers. This mechanism can be explained by the fact that at low Rayleigh number regime, there is no flow in the region below $y = A$ on the left-hand wall. The height of the active left wall essentially remains the same even when the phase shift is changed from 0 to $\delta/2$, so heat transfer rate has little effect. For the cases with higher Rayleigh numbers, the streamlines penetrate the roughness element area adjacent to the left wall, so heat transfer occurs from

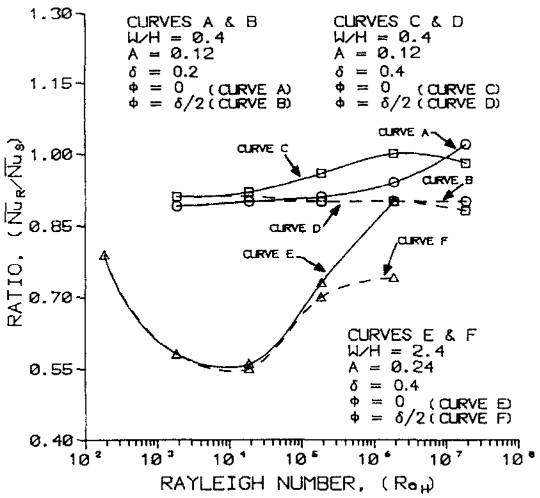

FIG. 10. The effect of phase shift on the ratio of Nusselt numbers for an enclosure with $Pr = 4.52$.

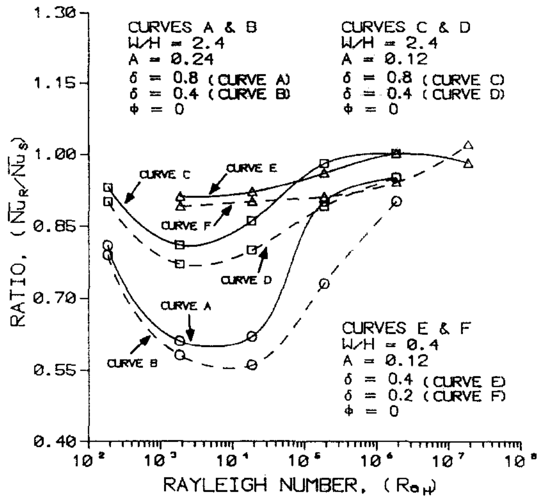

FIG. 12. The effect of period on the ratio of Nusselt numbers for an enclosure with $Pr = 4.52$.

the entire height of the left hot wall. However, when the phase shift is changed from 0 to $\delta/2$ (for high Rayleigh number cases), the active height of the hot wall is reduced and therefore the heat transfer rate is also reduced. The maximum reduction in the ratio is found to be 17.78% with $Pr = 4.52$ for an enclosure with $W/H = 2.4$, $A = 0.24$, and $\delta = 0.4$ (five elements) at $Ra_H = 1.85 \times 10^6$. On the other hand, for an enclosure with $W/H = 0.4$, $A = 0.12$, $Pr = 4.52$, and $Ra_H = 1.85 \times 10^7$, the decrements in the ratios are 11.76% when $\delta = 0.2$ (two elements) and 10.20% when $\delta = 0.4$ (one element).

To demonstrate the effect of the amplitude change, curve A is compared with curve B and curve C is compared with curve D in Fig. 11. It is observed that increasing the amplitude actually decreases the Nusselt number ratio, that is, more reduction in the heat transfer rate across the enclosure. The reason for

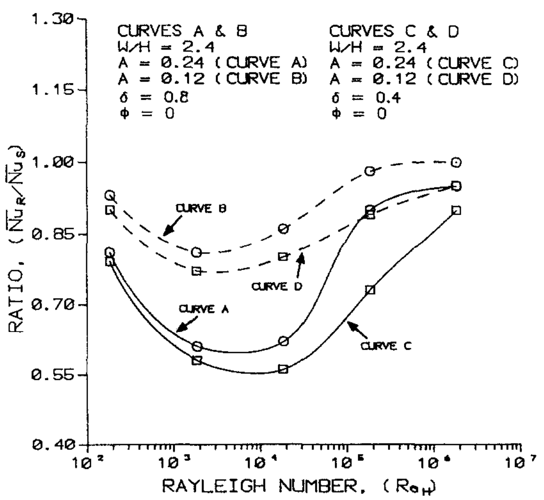

FIG. 11. The effect of amplitude on the ratio of Nusselt numbers for an enclosure with $Pr = 4.52$.

this is discussed later. In this study the maximum reduction in the Nusselt number ratio due to the increase of amplitude is found to be 30%. This is observed when the amplitude, A, is increased from 0.12 to 0.24 for an enclosure with $W/H = 2.4$, $\delta = 0.4$, $\phi = 0$, at $Pr = 4.52$ and $Ra_H = 1.85 \times 10^4$.

The effect of changing the period is determined by comparing curve A with curve B, curve C with curve D, and curve E with curve F in Fig. 12. From the general trend of this study, it is observed that decreasing the period (i.e. increasing the number of elements), decreases the Nusselt number ratio. The maximum reduction in the ratio due to the decrease in period is found to be 18.89%. This is observed when the period of an enclosure with $W/H = 2.4$, $A = 0.24$, $\phi = 0$ at $Pr = 4.52$ and $Ra_H = 1.85 \times 10^5$ is changed from 0.8 (three elements) to 0.4 (six elements). The reason for the decrease in the Nusselt number ratio, as observed in Figs. 11 and 12, due to the increase in amplitude and decrease in period will be discussed now. The Nusselt number ratios in these cases are decreased due to the fact that the vertical heat transfer surfaces suffer more restrictions to convective effects by roughness elements as amplitude increases or period decreases. To study the effect of Prandtl number, several cases were executed for an enclosure with $W/H = 2.4$, $\delta = 0.4$, $\phi = 0$. These results are shown in Fig. 13 where the dashed lines are for the cases with amplitude $(A) = 0.12$ and the solid lines are for the cases with amplitude $(A) = 0.24$. The circular and square symbols are for Prandtl numbers of 0.72 and 4.52, respectively. It can be seen from this figure that the ratio of Nusselt numbers has little or no effect with Prandtl number change.

It is worth discussing here the results of the present study and the results of refs. [1–3] because these studies are related. In ref. [1] the authors conclude that due to the presence of distributed roughness elements on the hot wall, an earlier transition of the boundary

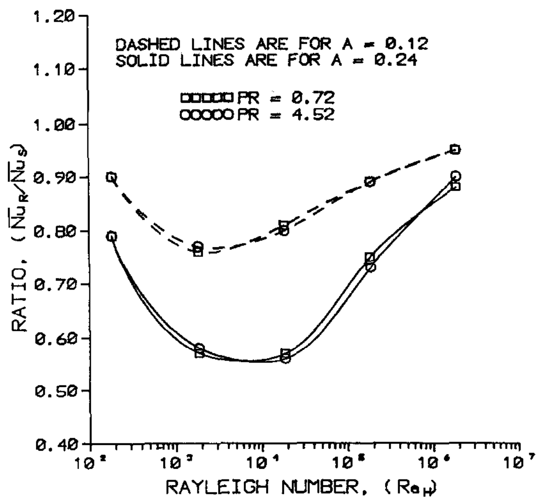

FIG. 13. The effect of Prandtl number on the ratio of Nusselt numbers for an enclosure with $W/H = 2.4$, $\delta = 0.4$, $\phi = 0$.

layer occurs so heat transfer enhancement is observed. In refs. [2, 3] the roughness elements were not distributed, rather one or two roughness elements were placed on the hot wall of the enclosure. The authors of refs. [2, 3] reported that in some cases the roughness elements contributed to the heat transfer augmentation. They concluded that the roughness elements increased the surface area of the hot wall and that the fluid velocity increased in parts of the roughness elements to overcome the obstruction. These two effects contributed to the heat transfer augmentation. However, the authors also concluded that in some cases heat transfer rate did not increase because part of the roughness element also suffered a decrease in velocity and, as such, a decrease in heat transfer there. This effect counterbalanced the two effects mentioned earlier and the overall effect was no heat transfer enhancement. In the present research the roughness elements were mounted on the bottom adiabatic wall and a decrement in heat transfer rate across the enclosure occurred. This is due to the fact that the roughness elements in this study actually shield the convective effects from the vertical walls, so decrement in heat transfer is observed.

As mentioned earlier, the roughness elements were mounted on the bottom horizontal wall of the enclosure in such a manner that the volume of the fluid in the rough-walled enclosure remained the same as that of the corresponding smooth-walled enclosure. If this design consideration was not taken into account, and the roughness elements were merely mounted on the wall of the smooth-walled enclosure, then the volume of the fluid in the enclosure would be reduced. In that case it would be impossible to determine whether the alteration of flow in the rough-walled enclosure was due to the presence of the roughness elements, or due to the decrease of the amount of fluid, or both. To explore this design criteria, five cases were executed for an enclosure with $W/H = 2.4$, $A = 0.24$, $\delta = 0.4$, $\phi = 0$. The roughness elements were mounted on the bottom adiabatic wall of a smooth-walled enclosure in such a manner that the volume of the fluid in the enclosure is reduced by 24%. The streamlines and isotherms for the flow in such an enclosure at $Pr = 4.52$ and $Ra_H = 1.85 \times 10^6$ are shown in Fig. 14. The arrows show the location of the corresponding smooth-walled enclosure. The flow in this enclosure should be compared with the flow of the enclosure shown in Fig. 7, where the volume of the fluid is the same as that of the corresponding smooth-walled enclosure. The ratio of Nusselt numbers for the flow in Fig. 7 is 0.90 (i.e. 10% reduction in heat transfer). On the other hand, the ratio of Nusselt numbers for the flow in Fig. 14 is 0.62 (i.e. 38% reduction in heat transfer). This shows that for the case of flow shown in Fig. 14 (reduced volume) the ratio of Nusselt numbers is reduced by 31.11% from the ratio of Nusselt numbers for the flow shown in Fig. 7 (same volume). In the present research, the maximum reduction in the ratio due to the effect of reduced volume of fluid is found to be 69.64% at $Ra_H = 1.85 \times 10^6$ and $Pr = 4.52$. The comparison of these ratios of Nusselt numbers at other Rayleigh numbers is shown in Fig. 15.

CONCLUSIONS

Based on the parametric study in this research, the following conclusions are drawn.

(1) The mounting of a periodic array of large rectangular roughness elements on the bottom adiabatic wall of the enclosure reduces the heat transfer rate across the enclosure. The reduction is more for enclosures with W/H (aspect ratio) > 1. The highest value of heat transfer reduction obtained in this research is 44%, obtained for an enclosure with $W/H = 2.4$ at $Ra_H = 1.85 \times 10^4$ and $Pr = 4.52$.

(2) The ratio of Nusselt numbers $[\overline{Nu}_R/\overline{Nu}_S]$ increases with the increase of Rayleigh numbers for enclosures with $W/H \leqslant 1$. For an enclosure with $W/H > 1$, the ratio at first decreases to a certain value and then increases with the increase of Rayleigh numbers.

(3) Little or no changes occur in the ratio of Nusselt numbers due to the change of phase shift in the conduction regime. However, the ratio decreases in the transition and boundary-layer regimes, when the phase shift (ϕ) is changed from 0 to $\delta/2$.

(4) With the increase of the amplitude of the roughness elements, the reduction in heat transfer rate increases.

(5) The reduction of the heat transfer rate increases by the reduction of the period of the roughness elements (i.e. increasing the number of elements).

(6) The Prandtl number has little or no effect on the ratio of Nusselt numbers.

(7) If the roughness elements are added in a manner so that the volume of the fluid in the enclosure is

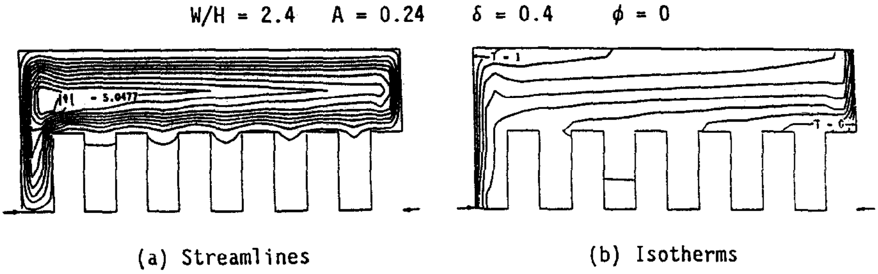

FIG. 14. Computed streamlines and isotherms in a rough-walled enclosure with 24% reduction in volume of the fluid due to the addition of the roughness elements, at $Ra_H = 1.85 \times 10^6$, $Pr = 4.52$.

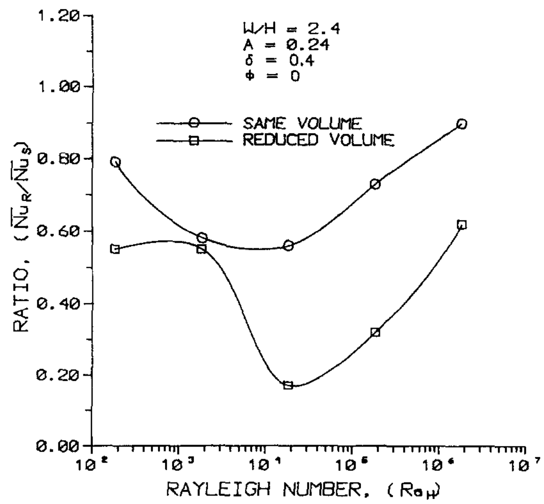

FIG. 15. The effect of adding the roughness elements in a manner such that the volume of the fluid in the enclosure is reduced by 24%, with $Pr = 4.52$.

reduced, significant reduction of the ratio of Nusselt numbers occurs.

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EFFET DE LA RUGOSITE D'UNE PAROI ADIABATIQUE SUR LA CONVECTION THERMIQUE NATURELLE DANS DES CAVITES VERTICALES

Résumé—On étudie numériquement la convection thermique variable dans une cavité verticale bidimensionnelle contenant des arrangements périodiques d'éléments rectangulaires à la paroi inférieure horizontale. On considère les domaines de nombre de Rayleigh (Ra_H) et de rapport de forme (W/H) respectivement de $1{,}85\times10^2$–$1{,}85\times10^7$ et 0,4–2,4 ainsi que des nombres de Prandtl de 0,72 et 4,52. On trouve que les éléments de grosse rugosité réduisent le transfert thermique à travers la cavité, la réduction est plus significative pour les cavités avec $W/H > 1$. La valeur de réduction la plus forte obtenue est de 44% ; elle correspond à $W/H = 2{,}4$ $Ra_H = 1{,}85\times10^4$ et $Pr = 4{,}52$.

DER EINFLUSS ADIABATER WANDRAUHIGKEITSELEMENTE AUF DEN WÄRMEÜBERGANG BEI NATÜRLICHER KONVEKTION IN SENKRECHTEN HOHLRÄUMEN

Zusammenfassung—Der Wärmeübergang bei natürlicher Konvektion in einem zweidimensionalen senkrechten Hohlraum wird numerisch untersucht. Dies geschieht für den Fall, daß die waagerechte untere Seite des Hohlraums periodische Anordnungen von großen rechteckigen Elementen trägt. Die numerischen Untersuchungen werden für folgende Parameterbereiche durchgeführt: Rayleigh-Zahl ($1{,}85\times10^2 < Ra_H < 1{,}85\times10^7$), Seitenverhältnis ($0{,}4 < W/H < 2{,}4$) und Prandtl-Zahl ($Pr = 0{,}72$ und 4,52). Es zeigt sich, daß die großen Rauhigkeitselemente den Wärmetransport im Hohlraum behindern. Die Verringerung des Wärmetransports ist in Hohlräumen mit $W/H > 1$ deutlicher. Die stärkste, in der vorliegenden Untersuchung berechnete Verringerung des Wärmeübergangs beträgt 44%. Dieser Wert ergibt sich für einen Hohlraum mit $W/H = 2{,}4$ bei $Ra_H = 1{,}85\times10^4$ und $Pr = 4{,}52$.

ВЛИЯНИЕ ШЕРОХОВАТОСТИ АДИАБАТИЧЕСКОЙ СТЕНКИ НА ЕСТЕСТВЕННОКОНВЕКТИВНЫЙ ТЕПЛОПЕРЕНОС В ВЕРТИКАЛЬНЫХ ПОЛОСТЯХ

Аннотация—Численно исследуется естественноконвективный теплоперенос в двумерной вертикальной полости с периодическими рядами крупных прямоугольных элементов на нижней горизонтальной стенке. Исследование проводится в диапазонах изменения числа Рэлея (Ra_H) и отношения сторон (W/H), составляющих соответственно $1{,}85\times10^2$–$1{,}85\times10^7$ и 0,4–2,4. Рассматриваются числа Прандтля (Pr), равные 0.72 и 4,52. Найдено, что наличие крупных элементов шероховатости приводит к уменьшению скорости теплопереноса в полости, причем это уменьшение более значительно в полостях с $W/H > 1$. Максимальная величина уменьшения теплопереноса составляет 44% в случае полости с $W/H = 2{,}4$ при $Ra_H = 1{,}85\times10^4$ и $Pr = 4{,}52$.